Discrete Talbot Effect in Waveguide Arrays

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We report the first observation of discrete Talbot revivals in one-dimensional waveguide arrays. Unlike continuous systems where the Talbot self-imaging effect always occurs irrespective of the pattern period, in discrete configurations this process is only possible for a specific set of periodicities. Recurrence of different input periodic patterns is observed in good agreement with theory.

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The repeated self-imaging of a diffraction grating was first observed by Talbot [1] in 1836. A few decades later Rayleigh explained this remarkable effect by showing that any periodic one-dimensional field pattern reappears, upon propagation, at even integer multiples of the so-called Talbot distance $z_T = D^2/\lambda$, where D represents the spatial period of the pattern and λ the light wavelength [2]. This process, being a direct result of Fresnel diffraction, is among the most basic phenomena in optics [3–7]. In addition to the integer Talbot effect, fractional revivals are also known to occur at distances that are rational multiples of z_T, i.e., $z/z_T = p/q$, where p and q are relatively prime integers. In fact, as shown in several studies, these fractional Talbot images consist of q coherently superimposed and equally spaced copies of the initial image [3,5,6]. On the other hand, if z/z_T happens to be irrational, the resulting image is fractal in nature [5]. This interesting relationship between the Talbot effect and number theoretic issues has been recently suggested as a possible means to factorize integers [8].

In the last few years, the interest in optical Talbot effects has been renewed because of possible applications, not only in the spatial [9] but also in the temporal domain (in optical dispersive fibers) [10]. In addition to optics, Talbot recurrences have been encountered in many other areas of physics, such as in atom optics [11], Bose-Einstein condensates [12], and in the interferometry of large C_{70} fullerene molecules [13]. Generally speaking, the Talbot process belongs to a broader family of phenomena exhibiting wave packet revivals [14]. Revivals of this sort can occur, for example, in the generation and detection of atomic Rydberg electron wave packets [15], in molecular systems [16], in quantum billiards and carpets [14,17], during Bloch oscillations [18], and in systems described by the Jaynes-Cummings model [14], just to mention a few. Yet, so far, the Talbot process has only been investigated in continuous systems using either the Fresnel equation (in optics) or its mathematically equivalent Schrödinger equation in atom optics.

Quite recently there has been considerable interest in wave propagation phenomena in discrete structures [19]. As in solid state physics, such optical discrete or lattice configurations are known to exhibit a succession of allowed Floquet-Bloch bands and forbidden band gaps. In weakly coupled systems, the Floquet-Bloch states can be accurately described by local modes and thus the tight-binding approximation is applicable [20]. As a result, the field evolution equation becomes effectively discretized. In optics, arrays of evanescently coupled waveguides [19] or chains of coupled microresonators [21] are prime examples of such structures where discrete wave dynamics can be observed and investigated. Unlike in the bulk, where the dispersion relation (of the paraxial Fresnel or Schrödinger equation) is parabolic, in lattices the dispersion curve has a cosinelike character. This in turn can lead to altogether new phenomena (for example, zero or anomalous diffraction) that have no analogue in the bulk [22]. The question naturally arises as to whether the Talbot effect is also possible in discrete systems. And if so, how does it differ from that occurring in continuous systems?

In this Letter we report the experimental observation of discrete Talbot effects in weakly coupled waveguide arrays. Our theoretical analysis indicates that Talbot recurrences occur only when the period N of the initial pattern belongs to the set $N \in \{1, 2, 3, 4, 6\}$. This is unlike what occurs in the continuous Talbot process where the revivals are period independent. Our experimental results were found to be in good agreement with theoretical predictions.

To analyze the discrete Talbot effect, let us hypothetically consider an infinite array of weakly coupled discrete elements. Depending on the nature of the array, these elements may be, for example, optical waveguides or coupled microcavities. In such arrangements the modal electric field amplitudes evolve according to [19,21]

$$i\frac{dU_n}{d\xi} + \kappa(U_{n+1} + U_{n-1}) = 0, \qquad (1)$$

where κ stands for the coupling coefficient between elements, and the variable ξ may represent either distance (as in the case of waveguide arrays) or time if microcavities are involved. Equation (1) is known to admit periodic Floquet-Bloch–like solutions, of the form $U_n = \exp(inQ)\exp(i\lambda\xi)$, where Q is a phase-shift among successive sites and λ is an eigenvalue, given by $\lambda = 2\kappa\cos(Q)$. For the Talbot effect to take place, the input field distribution should be periodic, and thus in general $U_{n+N} = U_n$, where N represents the spatial period of the input. Because of this periodic boundary condition, Q can take values only from the discrete set $Q_m = m\theta$, where $\theta = 2\pi/N$ and $m = 0, 1, 2, \ldots, N-1$. Therefore, as a result of periodicity, the field evolution at site n can be in general described through the orthonormal set of functions $a_n^{(m)} = N^{-1/2}\exp(inQ_m)\exp(i\lambda_m\xi)$, i.e., $U_n^{(N)} = \Sigma_{m=0}^{N-1} c_m a_n^{(m)}$. It is therefore clear that field revivals are possible at intervals $\hat{\xi}$ if $\lambda_j\hat{\xi} = 2\nu\pi$ (where ν is an integer), and hence the ratio of any two eigenvalues must be a rational number; i.e., $\lambda_i/\lambda_j = p/q$, where p and q are relatively prime integers. From the ratio λ_1/λ_0, one arrives at the conclusion that $\cos(2\pi/N)$ must also be rational for field revivals to occur. In addition, one can also directly show that the intensity patterns will also repeat if the ratio $(\lambda_\mu - \lambda_\nu)/(\lambda_i - \lambda_j) = p/q$ is also rational, where the eigenvalue indices $\mu, \nu, i, j \in \{0, 1, \ldots, N-1\}$ and are taken at least three at a time. By considering the ratio $(\lambda_2 - \lambda_0)/(\lambda_1 - \lambda_0)$, one can then reestablish the fact that $\cos(2\pi/N)$ should be rational. Therefore, for discrete Talbot revivals (field or intensity) to occur, it is necessary that $\cos(2\pi/N) = p/q$; i.e., it is a rational number. The question now is this: For which values of N is $\cos(2\pi/N)$ a rational number? We address this issue by first observing the fact that all higher-order eigenvalues can be obtained from the first one, using Chebyshev polynomials $T_m(x)$; that is, $\cos(m\theta) = T_m(\cos(\theta))$, where $\theta = 2\pi/N$, and $T_m(x) = \Sigma_{k=0}^{[m]} c_k^{(m)} x^{m-2k}$, where $[m]$ represents the integer part of m. We note that all the Chebyshev coefficients $c_k^{(m)}$ are integer numbers and, of importance to this discussion, is the fact that the first Chebyshev coefficient is given by $c_0^{(m)} = 2^{m-1}$. Given that $c_k^{(m)}$ are integers, then $\cos(2m\pi/N)$ is rational if and only if $\cos(2\pi/N)$ is rational. To find all possible N's that will permit Talbot recurrences, we first assume that N is odd. From the relation $T_N(\cos(\theta)) = \cos(N\theta) = 1$, one obtains the following polynomial in $\cos(\theta)$: $2^{N-1}(\cos(\theta))^N + \cdots + c_{[N/2]}^N \cos(\theta) - 1 = 0$. By applying the rational root theorem, the possible rational roots of this polynomial, if any, should belong to the set $\pm\{1, 1/2, 1/2^2, \ldots, 1/2^{N-1}\}$. It turns out that these are indeed roots only if $N = 1, 3$. For $N = 5$, $\cos(2\pi/5) = (\sqrt{5} - 1)/4$ is irrational, and in addition for any odd integer N greater than 6, we expect that $1/2 < \cos(2\pi/N) < 1$. Since from the previous discussion this is impossible, then $\cos(2\pi/N)$ is rational only if $N = 1, 3$. By using similar techniques and the fundamental theorem of arithmetic (unique factorization theorem), one can then show that for even N, $\cos(2\pi/N)$ is rational only if $N = 2, 4, 6$. Therefore, strictly speaking, discrete Talbot revivals are possible only for a finite set of periodicities $N \in \{1, 2, 3, 4, 6\}$, where $N = 1$ represents the trivial case of a discrete plane-wave solution. For any other periodicity in general, the field evolution is nonperiodic. This is in contrast to what happens in the continuous Talbot case where the recurrences happen to be period independent. Of course, for specific periodic inputs, it is also possible to have revivals even when N does not belong to the above mentioned set. This may happen in cases where only a subset of eigenvalues is involved (because of the input pattern) that happen to be rational with respect to each other.

We will now illustrate some of the aspects associated with the discrete Talbot effect by means of relevant examples. Let us first assume a binary pattern at the input. More specifically, let $U_n = a_0\exp(in\phi)$ for even n sites and $U_n = b_0\exp(in\phi)$ for odd. In this case, one can show that the field in the even or odd elements evolves according to $U_n = [(a_0, b_0)\cos(2\kappa\xi\cos(\phi)) + i(b_0, a_0) \times \sin(2\kappa\xi\cos(\phi))]\exp(in\phi)$. Figure 1(a) depicts periodic intensity revivals when the binary input is $\{1, 0, 1, 0, \ldots\}$. The intensity Talbot "carpet" corresponding to this case is shown in units of coupling lengths $L_c = \pi/2\kappa$. For this example where $b_0 = \phi = 0$, the patterns reappear every L_c; i.e., the discrete Talbot distance is $\xi_T = L_c$. An interesting case arises when the array is excited at an angle (at a finite Bloch momentum [19]) and thus the binary input is phase shifted according to $\{e^{i\phi}, 0, e^{3i\phi}, 0, \ldots\}$ ($a_0 = 1$, $b_0 = 0$). From the previous discussion, one then finds that this latter case, the discrete Talbot period, is given by $\xi_T = L_c/\cos(\phi)$. It is interesting to note that as ϕ approaches $\pi/2$ the Talbot revivals slow down and totally disappear at $\phi = \pi/2$ (i.e., when the input pattern is $\{1, 0, -1, 0, \ldots\}$), as shown in Fig. 1(b). One may interpret this effect from the fact that the diffraction or dispersion of the array is zero in the middle of the Brillouin zone (at $\pi/2$) and so the Talbot process that derives from these effects vanishes. Figure 1(c) shows another Talbot intensity carpet when the input periodic pattern $\{1, 0, 0, 1, 0, 0, \ldots\}$ has a period $N = 3$. In this case the intensity in the initially excited channels evolves according to $(5 + 4\cos(3\kappa\xi))/9$, whereas in the unexcited channels it varies like $(2/9)(1 - \cos(3\kappa\xi))$. For this pattern the Talbot period is given by $\xi_T = 4L_c/3$. In general, for $N \in \{1, 2, 3, 4, 6\}$ and for in-phase excitations, the Talbot recurrence distance is given by the largest period $\xi_T = 2\pi/(\lambda_i - \lambda_j)$ that results from the eigenvalues involved in the initial pattern.

To experimentally demonstrate discrete Talbot effects, we used a channel waveguide array consisting of 101 guides. This array was fabricated on 70 mm long Z-cut $LiNbO_3$ wafer using standard lithography and titanium in-diffusion techniques [23]. The center-to-center spacing between the array channels was 15 μm. The interchannel coupling length was measured experimentally as a function

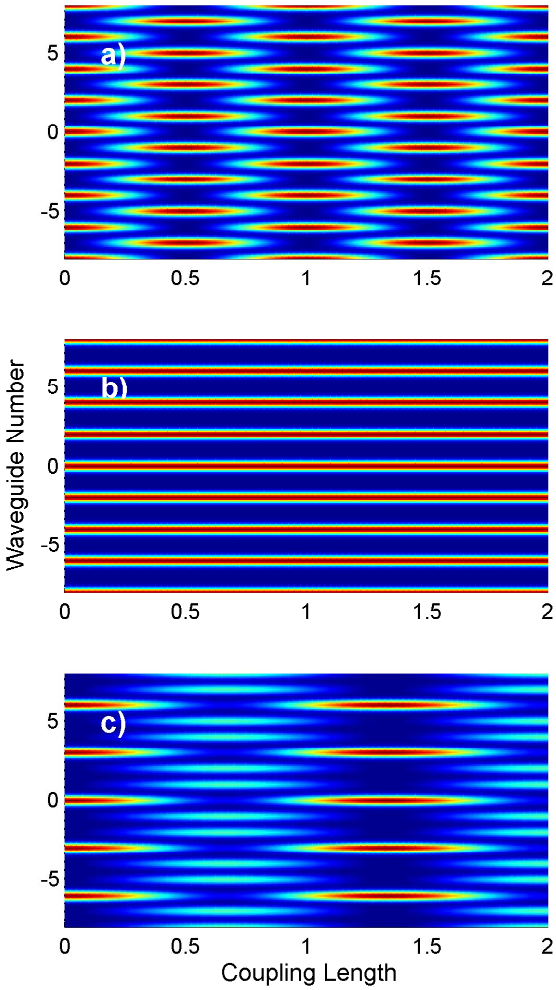

FIG. 1 (color online). Talbot intensity ''carpets'' for different input field patterns: (a) $\{1, 0, 1, 0, \ldots\}$; (b) $\{1, 0, -1, 0, \ldots\}$; (c) $\{1, 0, 0, 1, 0, 0, \ldots\}$.

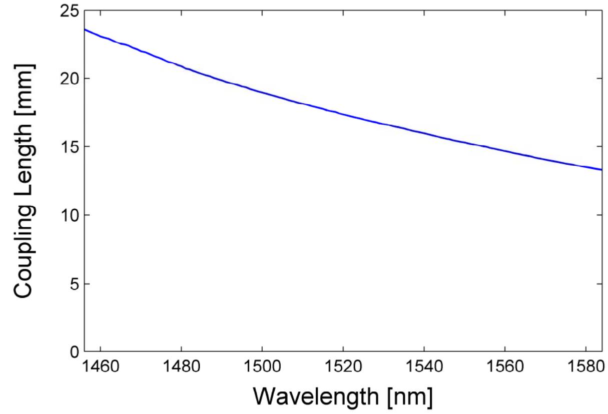

FIG. 2 (color online). Coupling length of the array used in the experiment as a function of wavelength.

of wavelength (as shown in Fig. 2) by fitting the diffraction pattern arising from the excitation of a single waveguide to that expected from theory; i.e., $U_n(\xi) = (i)^n J_n(2\kappa\xi)$ [24], where $J_n(x)$ represents a Bessel function of the nth order. In addition, using the beam deflection scheme, we experimentally probed the dispersion relation (dk_z/dk_x vs k_x) of the array. The displacement curve was found to be sinusoidal, which justifies the use of the tight-binding approximation (or coupled-mode theory) in Eq. (1).

In the experimental setup shown in Fig. 3 we used a HP81680 tunable diode laser. The beam was shaped, using a telescope, to be highly elliptical (500×3.5 μm full width at half of maximum) and was focused by a 10× microscope objective onto the input facet of the array sample. Amplitude transmission masks, with periodicities that are multiples of the array interchannel spacing and exhibiting different patterns, were fabricated using laser writing and etching techniques. The masks were then put in contact with the sample for clean in-phase mode excitation. To control the tilt of the input beam and hence the initial phase difference between adjacent channels, a mirror on a motorized stage was placed between the telescope and the microscope objective. Because of the sample's excellent linear properties (low scattering), we were not able to observe the Talbot revivals when looking from the top. Instead, an indirect observation of the Talbot process at the output of the array was possible by varying the wavelength (and hence the coupling length) over the full spectral range of the laser (1456–1584 nm). This change in coupling strength with wavelength is essentially equivalent to varying the effective sample length. This in turn allows one to observe the Talbot effect without affecting the diffraction properties of the beam.

The experimental results corresponding to the periodic $\{1, 0, 1, 0, \ldots\}$, $\{1, 0, -1, 0, \ldots\}$, and $\{1, 0, 0, 1, 0, 0, \ldots\}$ excitation conditions (simulated in Fig. 1) are shown in Figs. 4(a)–4(c), respectively. These figures depict the intensity at the output of the array as a function of wavelength, in good agreement with theory. In Fig. 4(a), we observe a Talbot recurrence and in between an intermediate state [at $(1, 3)L_c/4$] where all elements are equally excited. On the other hand, as per our previous discussion, in Fig. 4(b) this periodic recursion disappears since the phase difference between successive waveguides is $\pi/2$. Similarly, Fig. 4(c) demonstrates Talbot revivals when the

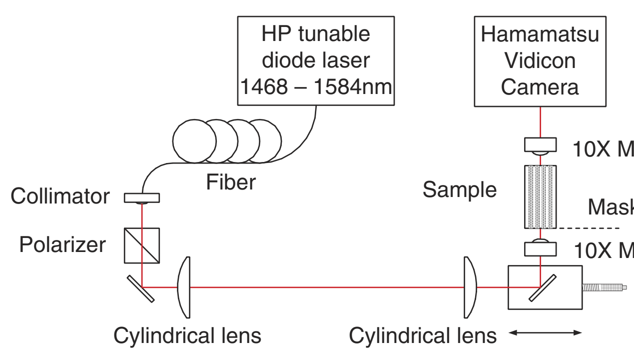

FIG. 3 (color online). Experimental setup.

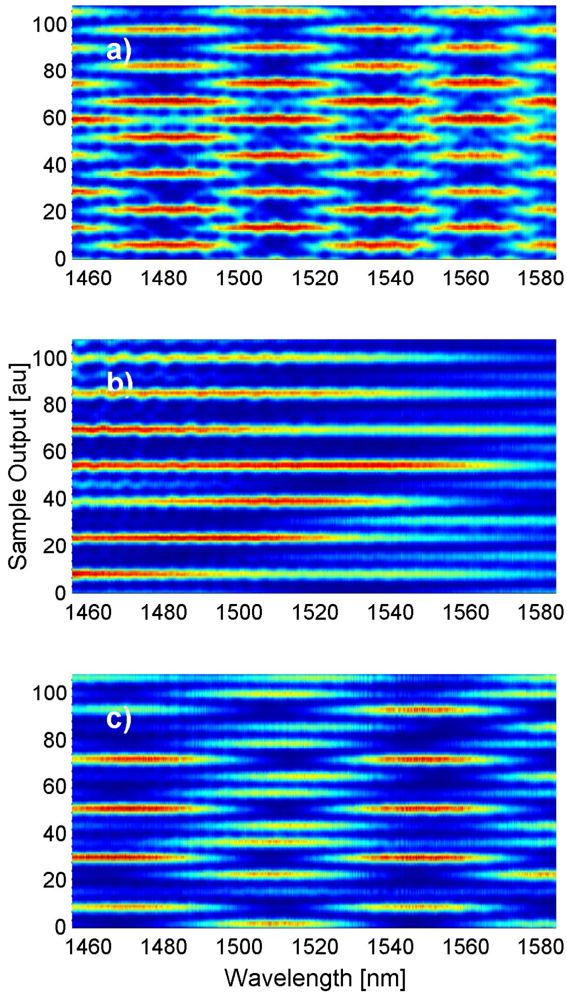

FIG. 4 (color online). Experimentally observed Talbot revivals for (a) $\{1, 0, 1, 0, \ldots\}$, (b) $\{1, 0, -1, 0, \ldots\}$, and (c) $\{1, 0, 0, 1, 0, 0, \ldots\}$ field pattern excitations.

initial pattern has a period $N = 3$. The "wavy" nature of the observed patterns is a consequence of the wavelength tuning. This introduces wave front aberrations at the input facet due to the chromatic dispersion in the optical elements that leads to focal point shifts with wavelength. This results in a very weak excitation of higher-order bands which interfere with the lowest order band of interest. Additional experiments were also performed for the excitation conditions $\{1, 0, 0, 0, 1, 0, 0, 0, \ldots\}$, and again very good agreement with theory was obtained.

In conclusion, we have demonstrated, for the first time, discrete Talbot revivals in one-dimensional waveguide arrays. Unlike continuous systems where the Talbot self-imaging effect always occurs irrespective of the pattern period, in discrete configurations this process is only possible for a specific set of periodicities. Before closing, we would like to note that our results may be relevant to other areas of physics where such Talbot recurrences may be observed. These include, for example, optical wave propagation in photonic crystal structures and Bose-Einstein condensates in optically induced periodic potentials [14].

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